

# Modelling the influence of cracking and healing on modal properties of concrete beams

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## ABSTRACT

Concrete structures are commonly cracked when in service. To overcome issues arising from cracking, self-healing concrete is being developed. Together with the development of the material, techniques to verify and quantify self-healing are being developed. A number of destructive techniques have been used in the past. It would be beneficial to use non-destructive testing for continuous monitoring of self-healing. It is well known that cracking causes changes in natural frequencies of structural elements. A change in natural frequencies can be, therefore, used as an indication of damage. Consequently, the recovery of natural frequencies can be used as an indication of healing.

In this work, a model which enables calculation of mode shapes and natural frequencies is presented. First, it is shown that the proposed 2D model can reasonably mimic modal behavior of thin structures under different boundary conditions [6]. It is then used to assess the influence of cracking and healing on the change of modal properties of concrete beams. Finally, based on the modelling results, conclusions are drawn.

## 1. INTRODUCTION

Concrete structures always contain cracks of different sizes. However, if cracks are too big, they can be detrimental to durability or serviceability of a structure. In order to overcome this issue, a number of methods for self-healing of concrete were developed. Consequently, different experimental techniques have been used in order to verify and quantify self-healing [1]. Most of these techniques are, however, destructive. In reality, it would be beneficial to use non-destructive testing for continuous monitoring of self-healing.

It has been long known that damage and cracking cause changes in natural frequencies of structures [2]. A change in natural frequencies can be, therefore, used as an indication of damage. Consequently, the recovery of natural frequencies can be used as an indication of healing. A deeper understanding of these processes can be achieved through numerical modelling.

Spectral element method (SEM) has been used in recent years for dynamic analysis of structures [3,4]. It has shown to be suitable for modelling dynamic and modal behaviour of frame structures and periodic materials. On the other hand, Delft lattice model has proven to be suitable for modelling fracture and healing in cement based materials [1]. In this work, these two models are combined in order to study the influence on cracking and healing on modal properties of concrete beams.

## 2. METHOD

### 2.1. Modelling approach

The SEM is a method for dynamic analysis which is based on elements whose stiffness matrices are obtained by solving the governing differential equations in the frequency domain. The element shape functions are based on the solution of the equation of motion, and, therefore, the displacement field within each element is an exact solution, and not an approximation. This is in contrast to the FEM, where usually polynomial shape functions are used to approximate the displacement field. As a result, each geometrically and materially uniform structural element can be represented by a single spectral element [3-5], greatly reducing the total number of elements in the analysis. In the SEM, the element shape functions (and element stiffness matrices) are frequency-dependent, and the analysis is performed in the frequency domain.

The lattice model is used herein. The material is discretized as a set of spectral beam elements (Fig. 1). Therefore, the plate is modelled using simple beam elements with three degrees of freedom per node, two translations and one rotation (see Fig. 1). The plate is divided into a number of quadratic cells with a unit length of  $h$ , and a node is placed within each of these cells. These are the end nodes of lattice beam elements. A regular quadrangular lattice is used, connecting nodes in neighbouring cells (Fig. 1). More details are given in [5].

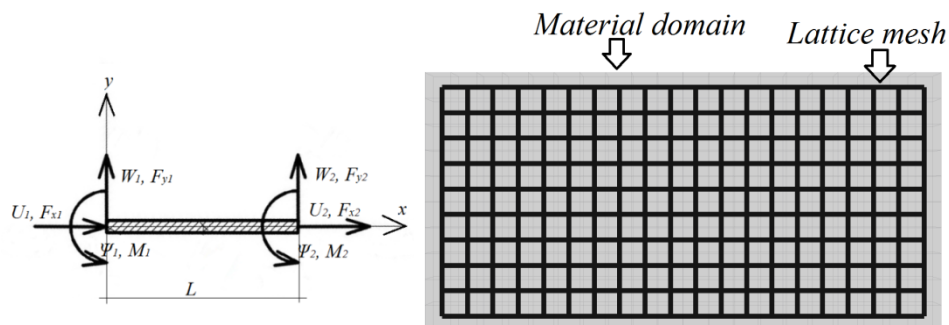


Figure 1: (Left) A beam element in a local coordinate system; (Right) Discretization of the material domain by a lattice mesh

### 2.2. Model verification

In their work, Bardell et al. [6] used the Rayleigh-Ritz method to study in-plane vibration of isotropic rectangular plates with varying boundary conditions. Apart from the modal frequencies, modal shapes for different test cases are provided. As the methodology presented in the current paper is quite new, it is necessary to test both the accuracy of the simple lattice model in terms of natural frequencies and its ability to reproduce correct modes of vibration. Therefore, work of Bardell et al. [6] was selected as a benchmark for evaluation of the proposed model.

The proposed model was used to analyse a completely free rectangular plate with aspect ratio of  $a=b=2$ . The material properties used in the calculation were  $E=207$  GPa,  $\rho=7800$  kg/m<sup>3</sup> and  $\nu=0.3$ . A lattice cell of a unit cell size ( $h=1$ ) was used in all analyses. The calculated natural frequencies were transformed into a non-dimensional form for comparison with benchmark results. In figure 2, a comparison between results obtained by the lattice model and the benchmark results are given.

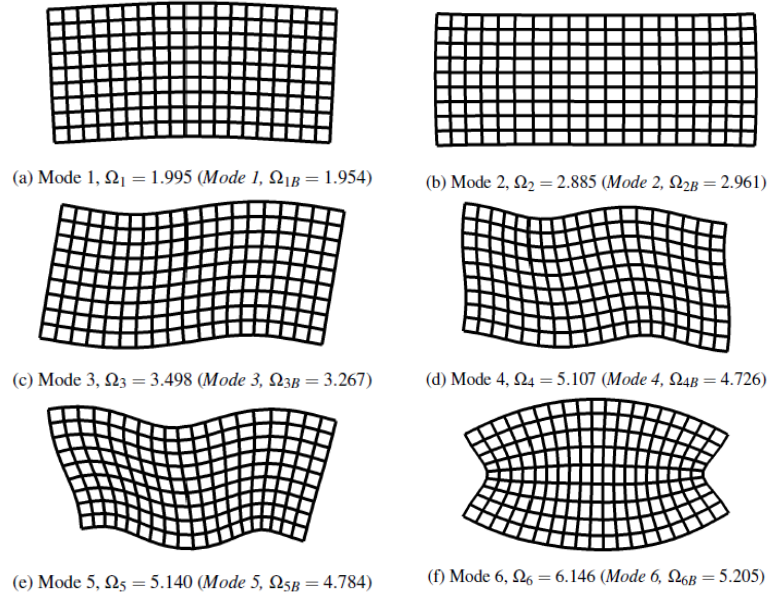


Figure 2: Modes and natural frequencies of a free isotropic plate,  $a = 2b$ . In parentheses, the corresponding mode and natural frequency reported in [6] is given

### 3. RESULTS AND DISCUSSION

Here, a modal analysis of a simply supported concrete beam ( $160 \times 40 \text{ mm}^2$ ) is given. It consists of spherical inclusion (aggregate) particles embedded in a mortar matrix. The model was first subjected to a simulated three-point bending test. Mechanical properties used in the analysis are given in table 1. Then, the natural frequencies of the cracked specimen are compared to the initial values (Table 2). Finally, the cracks are assumed to be healed using glue, and the recovery of natural frequencies is observed.

Table 1: Mechanical properties used in the analysis

Phase	Young's modulus (GPa)	Tensile strength (MPa)	Volumetric mass density ( $\text{kg/m}^3$ )
Mortar	25	4	2100
Aggregate	70	8	2600
Interface	15	2.5	2100
Glue	3.5	20	1060

Microcracks occurring prior to the peak load cause a very small decrease in the first natural frequency. Second and third mode are not affected. This is also related to the damage location, because here the stiffness loss is in the middle of the span, which causes a reduction in mode 1 [4]. Just prior to failure, the first three modes are all significantly reduced, indicating significant damage. After the healing event (assuming complete filling of all cracks with glue), the natural frequencies have recovered, except the first natural frequency, which is still somewhat lower compared to the initial one. The reason for this is that, although the glue has filled up the crack, it has a somewhat lower stiffness compared to the initial material (see Table 1). Due to the modal shape of mode 1 (which is similar to that shown in Figure 2a), this mode is particularly affected by loss of stiffness in the middle of the beam span.

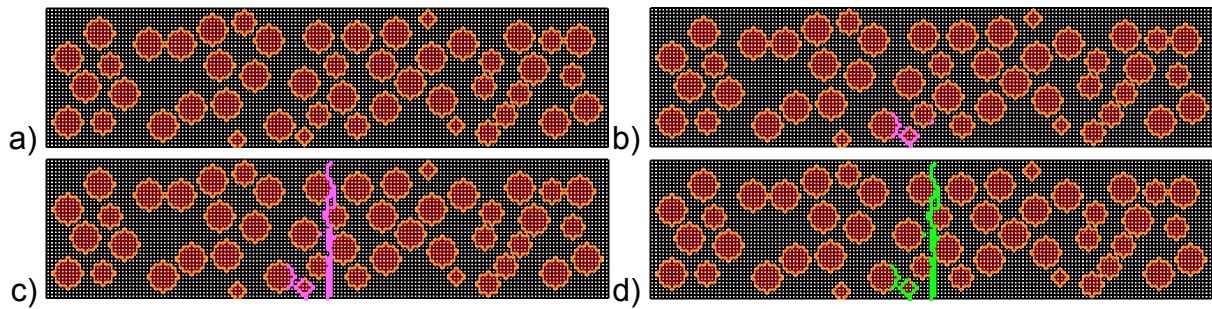


Figure 3: a) Material structure showing matrix (black), aggregates (brown) and the interface (light brown); b) cracks at peak; c) cracks prior to failure; d) healed cracks (b, c-pink – cracks; d-green – “healed” elements)

Table 2: Calculated natural frequencies for all analyses (Hz)

	Initial	Cracked (peak)	Cracked (pre-failure)	Healed
Mode 1	86	85	67	84
Mode 2	112	112	106	112
Mode 3	202	202	146	202

#### 4. CONCLUSIONS

In the current study, a model for studying in-plane vibration based on the lattice modelling framework is proposed. The model is validated using data from the literature. Finally, an example of a cracked and healed concrete beam is analysed. It was found that the model is able to detect changes in natural frequencies caused by cracking, and their subsequent recovery after crack healing. Although the model cannot be used in a quantitative sense due to its 2D nature, it can be used as an aid in non-destructive testing and monitoring of self-healing in concrete.

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